

# NAG C Library Function Document

## nag\_zstein (f08jxc)

### 1 Purpose

nag\_zstein (f08jxc) computes the eigenvectors of a real symmetric tridiagonal matrix corresponding to specified eigenvalues, by inverse iteration, storing the eigenvectors in a **complex** array.

### 2 Specification

```
void nag_zstein (Nag_OrderType order, Integer n, const double d[], const double e[],
                Integer m, const double w[], const Integer iblock[], const Integer isplit[],
                Complex z[], Integer pdz, Integer ifailv[], NagError *fail)
```

### 3 Description

nag\_zstein (f08jxc) computes the eigenvectors of a real symmetric tridiagonal matrix  $T$  corresponding to specified eigenvalues, by inverse iteration (see Jessup and Ipsen (1992)). It is designed to be used in particular after the specified eigenvalues have been computed by nag\_dstebz (f08jjc) with **order** = **Nag\_ByBlock**, but may also be used when the eigenvalues have been computed by other f08 or f02 functions.

The eigenvectors of  $T$  are real, but are stored by this function in a **complex** array. If  $T$  has been formed by reduction of a full complex Hermitian matrix  $A$  to tridiagonal form, then eigenvectors of  $T$  may be transformed to (complex) eigenvectors of  $A$ , by a call to nag\_zunmtr (f08fuc) or nag\_zupmtr (f08guc).

nag\_dstebz (f08jjc) determines whether the matrix  $T$  splits into block diagonal form:

$$T = \begin{pmatrix} T_1 & & & \\ & T_2 & & \\ & & \ddots & \\ & & & T_p \end{pmatrix}$$

and passes details of the block structure to this function in the arrays **iblock** and **isplit**. This function can then take advantage of the block structure by performing inverse iteration on each block  $T_i$  separately, which is more efficient than using the whole matrix.

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Jessup E and Ipsen I C F (1992) Improving the accuracy of inverse iteration *SIAM J. Sci. Statist. Comput.* **13** 550–572

### 5 Parameters

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag\_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

*Constraint:* **order** = **Nag\_RowMajor** or **Nag\_ColMajor**.

- 2: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $T$ .  
*Constraint:*  $n \geq 0$ .
- 3: **d**[ $dim$ ] – const double *Input*  
**Note:** the dimension,  $dim$ , of the array **d** must be at least  $\max(1, n)$ .  
*On entry:* the diagonal elements of the tridiagonal matrix  $T$ .
- 4: **e**[ $dim$ ] – const double *Input*  
**Note:** the dimension,  $dim$ , of the array **e** must be at least  $\max(1, n - 1)$ .  
*On entry:* the off-diagonal elements of the tridiagonal matrix  $T$ .
- 5: **m** – Integer *Input*  
*On entry:*  $m$ , the number of eigenvectors to be returned.  
*Constraint:*  $0 \leq m \leq n$ .
- 6: **w**[ $dim$ ] – const double *Input*  
**Note:** the dimension,  $dim$ , of the array **w** must be at least  $\max(1, n)$ .  
*On entry:* the eigenvalues of the tridiagonal matrix  $T$  stored in **w**[0] to **w**[ $m$ ], as returned by nag\_dstebz (f08jjc) with **rank** = **Nag\_ByBlock**. Eigenvalues associated with the first sub-matrix must be supplied first, in non-decreasing order; then those associated with the second sub-matrix, again in non-decreasing order; and so on.  
*Constraint:* if **iblock**[ $i$ ] = **iblock**[ $i + 1$ ], **w**[ $i$ ]  $\leq$  **w**[ $i + 1$ ] for  $i = 0, 1, \dots, m - 2$ .
- 7: **iblock**[ $dim$ ] – const Integer *Input*  
**Note:** the dimension,  $dim$ , of the array **iblock** must be at least  $\max(1, n)$ .  
*On entry:* the first  $m$  elements must contain the sub-matrix indices associated with the specified eigenvalues, as returned by nag\_dstebz (f08jjc) with **order** = **Nag\_ByBlock**. If the eigenvalues were not computed by nag\_dstebz (f08jjc) with **order** = **Nag\_ByBlock**, set **iblock**[ $i - 1$ ] to 1 for  $i = 1, 2, \dots, m$ .  
*Constraint:* **iblock**[ $i$ ]  $\leq$  **iblock**[ $i + 1$ ] for  $i = 0, 1, \dots, m - 2$ .
- 8: **isplit**[ $dim$ ] – const Integer *Input*  
**Note:** the dimension,  $dim$ , of the array **isplit** must be at least  $\max(1, n)$ .  
*On entry:* the points at which  $T$  breaks up into sub-matrices, as returned by nag\_dstebz (f08jjc) with **rank** = **Nag\_ByBlock**. If the eigenvalues were not computed by nag\_dstebz (f08jjc) with **rank** = **Nag\_ByBlock**, set **isplit**[0] to **n**.
- 9: **z**[ $dim$ ] – Complex *Output*  
**Note:** the dimension,  $dim$ , of the array **z** must be at least  $\max(1, \mathbf{pdz} \times \mathbf{m})$  when **order** = **Nag\_ColMajor** and at least  $\max(1, \mathbf{pdz} \times \mathbf{n})$  when **order** = **Nag\_RowMajor**.  
If **order** = **Nag\_ColMajor**, the  $(i, j)$ th element of the matrix  $Z$  is stored in **z**[( $j - 1$ )  $\times$  **pdz** +  $i - 1$ ] and if **order** = **Nag\_RowMajor**, the  $(i, j)$ th element of the matrix  $Z$  is stored in **z**[( $i - 1$ )  $\times$  **pdz** +  $j - 1$ ].  
*On exit:* the  $m$  eigenvectors, stored as columns of  $z$ ; the  $i$ th column corresponds to the  $i$ th specified eigenvalue, unless **fail** > 0 (in which case see Section 6).
- 10: **pdz** – Integer *Input*  
*On entry:* the stride separating matrix row or column elements (depending on the value of **order**) in the array **z**.

Constraints:

if **order** = **Nag\_ColMajor**, **pdz**  $\geq \max(1, \mathbf{n})$ ;  
 if **order** = **Nag\_RowMajor**, **pdz**  $\geq \max(1, \mathbf{m})$ .

11: **ifailv**[*dim*] – Integer

Output

**Note:** the dimension, *dim*, of the array **ifailv** must be at least  $\max(1, \mathbf{m})$ .

*On exit:* if **fail** = *i* > 0, the first *i* elements of **ifailv** contain the indices of any eigenvectors which have failed to converge. The rest of the first **m** elements of **ifailv** are set to 0.

12: **fail** – NagError \*

Output

The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_INT

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq 0$ .

On entry, **pdz** =  $\langle value \rangle$ .

Constraint: **pdz** > 0.

### NE\_INT\_2

On entry, **m** =  $\langle value \rangle$ , **n** =  $\langle value \rangle$ .

Constraint:  $0 \leq \mathbf{m} \leq \mathbf{n}$ .

On entry, **pdz** =  $\langle value \rangle$ , **n** =  $\langle value \rangle$ .

Constraint: **pdz**  $\geq \max(1, \mathbf{n})$ .

On entry, **pdz** =  $\langle value \rangle$ , **m** =  $\langle value \rangle$ .

Constraint: **pdz**  $\geq \max(1, \mathbf{m})$ .

### NE\_INT\_ARRAY

On entry, **iblock**[*i*]**w**[*i*]**iblock**[*i*] =  $\langle value \rangle$ .

Constraint: if **iblock**[*i*] = **iblock**[*i* + 1], **w**[*i*]  $\leq$  **w**[*i* + 1] for *i* = 0, ..., **m** - 2.

On entry, **iblock**[*i*]**w**[*i*]**iblock**[*i*] =  $\langle value \rangle$ .

Constraint: **iblock**[*i*]  $\leq$  **iblock**[*i* + 1] for *i* = 0, ..., **m** - 2.

### NE\_CONVERGENCE

$\langle value \rangle$  eigenvectors (as indicated by argument **ifailv**) each failed to converge in 5 iterations. The current iterate after 5 iterations is stored in the corresponding column of **z**.

### NE\_ALLOC\_FAIL

Memory allocation failed.

### NE\_BAD\_PARAM

On entry, parameter  $\langle value \rangle$  had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 7 Accuracy

Each computed eigenvector  $z_i$  is the exact eigenvector of a nearby matrix  $A + E_i$ , such that  $\|E_i\| = O(\epsilon)\|A\|$ , where  $\epsilon$  is the *machine precision*. Hence the residual is small:

$$\|Az_i - \lambda_i z_i\| = O(\epsilon)\|A\|.$$

However, a set of eigenvectors computed by this function may not be orthogonal to so high a degree of accuracy as those computed by nag\_zsteqr (f08jsc).

## 8 Further Comments

The real analogue of this function is nag\_dstein (f08jkc).

## 9 Example

See Section 9 of the document for nag\_zunmtr (f08fuc).

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